The 1st Annual West Windsor-Plainsboro Mathematics Expo

Saturday, October 26th, 2019

Grade 4 Problem Set

Directions:

Solve the following problems to the best of your ability. If you do not understand a problem or cannot solve it, skip it or ask for a hint. If you cannot solve a problem even after receiving all the hints for that problem, wait until the 30 minute mark and ask a proctor for further help or the solution. Some problems may not have hints.

Calculators are not allowed for these problems. You may, however, discuss with the people around you after 30 minutes have passed. That being said, do not ruin a problem for somebody by giving them a solution before they have a chance to attempt the problem themselves.

For this test, there will be 20 questions, and you will have a time limit of 60 minutes in total, which will be split into 30 minutes of individual work and 30 minutes of collaborative work. This test is very long and you are not expected to be able to do all of the problems. We recommend picking a range of 10-15 problems to work on. Please note that this is not a competition, and your goal is to enjoy the problems and gain experience.

HAVE FUN!

By the way, if you finish this exceptionally early, you are most likely an exceptional student. Thus, here is a slightly harder problem that you may wish to solve:

CHALLENGE: Pascal's Triangle is one of the most interesting triangles that was ever created. It is named after the famous mathematician, Blaise Pascal. It is not a geometric triangle, with 3 sides and 3 angles. Rather, it is an array of numbers that resembles a triangle: It can obviously continue on forever. The triangle may be

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}
$$

constructed in the following manner: In row 0 (the topmost row), there is a unique nonzero entry 1. Each entry of each subsequent row is constructed by adding the number above and to the left with the number above and to the right. This first row in Pascal's Triangle(the row with just one 1), is referred to as the 0th row, and all other rows after that follow. There are many interesting properties and patterns about Pascal's Triangle. Can you find some of them:

(a) What is the sum of all the numbers in the nth row of Pascal's Triangle?

(b) What is the kth number in the nth row of Pascal's Triangle? (Note that the first number in any row of Pascal's Triangle is referred to as the 0th number, and the rest follow from there).

(c) Consider the binomial expansion $(x+y)^n = a_0 \cdot x^n + a_1 \cdot x^{n-1} \cdot y + a_2 \cdot x^{n-2} \cdot y^2 + ... + a_{n-1} \cdot x \cdot y^{n-1} + a_n \cdot y^n$ Can you somehow relate $a_1, a_2 ... a_{n-1}, a_n$ with n? HINT: It involves k.

(d) Prove that the number of ways that Suraj can pick k pieces of candy from n total pieces of candy is simply the kth number of the nth row of Pascal's Triangle

(e) Prove that each row of Pascal's triangle gives the number of vertices at each distance from a fixed vertex in an n-dimensional cube. For example, in three dimensions, the third row (1 3 3 1) corresponds to the usual three-dimensional cube: fixing a vertex V, there is one vertex at distance 0 from V (that is, V itself), three vertices at distance 1, three vertices at distance $\sqrt{2}$ and one vertex at distance $\sqrt{3}$ (the vertex opposite V). The second row should define a square, the third row a cube, and other, larger dimensions should define multi-dimensional cubes.

1. What is $1 \cdot 1 + 2 \cdot 2$?

Solution: We can simply calculate each term simply, by using order of operations. Multiplication is higher than addition in PEMDAS(Parenthesis, Exponents, Multiplication, Division, Addition, and Subtraction), meaning we carry out that operation first. $1 \cdot 1 = 1$ and $2 \cdot 2 = 4$. We add these two quantities together, to get 5.

Hint: Think about order of operations!

2. The problem writer for this test was born on January 25th 2004. How old is he now (October 2019)?

Solution: It is the year 2019. He was born in 2004. 2019-2004 = 15. We have to make sure that he had his birthday this year, else he would not be 15. Indeed, January 25th 2004 has passed, so the problem writer is indeed 15.

Hint: Be sure to take into account whether or not he had his birthday this year!

3. What is 2/10 + 4/100 + 6/1000, expressed as a decimal?

Solution: 2/10 is simply 0.2, 4/100 is 0.04 and 6/1000 is 0.006. Lining these up by using column addition, we end up with our answer being 0.246.

Hint: Do not simplify the fractions. The question is asking for decimals, why not convert to decimals first?

4. Suraj likes to play basketball. If he makes 10 shots worth 2 points and 15 shots worth 3 points, how many total points does he score?

Solution: If he shot 10 shots worth 2 points, he scored 20 points off of shots worth 2 points. If he shot 15 shots worth 3 points, he scored 45 points off of shots worth 3 points. 20+45 = 65.

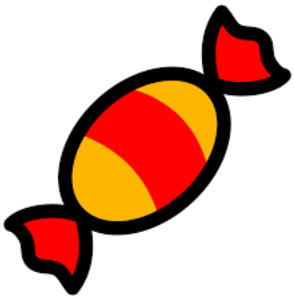
Hint: Remember, the total points scored is the number of shots you take, multiplied by the point value of that shot.

5. If $2x = 3x = 4x$, what is the value of x?

Solution: Clearly, there can only be one value of x that satisfies this system of equations. The only time where a number multiplied 2 times can equal that same number multiplied 3 times is if this number has special properties for multiplication. 0 seems to fit the description, making the answer 0.

Hint: Write a hint here: 2x can only equal 3x if x is a very special number. Can you figure out what this number is?

6. Henry has 300 pieces of candy. 243 of them are Kit-Kats. How many pieces of candy does Henry have that are not Kit-Kats?



Solution: 300-243 = 57.

Hint: Whenever you want a certain quantity, but you have the total number and the portion you don't want, consider subtraction!

7. Ansh is 6 feet 1 inch tall. If there are 12 inches in a foot, how tall is he in inches?

Solution: $12 \cdot 6 + 1 = 73$ inches tall.

Hint: First find how many feet tall Ansh is(this is given) and multiply with 12 to get the number of inches in those feet, then add to the remaining inches.

8. Allen has 20 dollars to spend at the bake sale. He wants to buy 4 brownies worth 2 dollars, 9 pieces of candy worth 1 dollar, and 2 key chains worth 50 cents. If Suraj steals 5 dollars from Allen, will he have enough to buy everything he wants?

Solution: The amount of dollars Allen needs to spend is 4(2) + 9(1) + 2(1/2) = 18. If Suraj stole 5 dollars form Allen, he would only have 15 dollars to spend. Allen does not have enough to spend, so the answer is no.

Hint: Try comparing the amount of money that Allen has left to the amount of money he wants to spend.

9. If a = -4, what is the largest number in the set 4a, 16/a, a+7 and -5/(a*a)

Solution: Let us calculate each of these values: 4a = 4(-4) = -16. 16/a = 16/-4 = -4. a+7 = -4+7 = 3. $-5/(4*4)$ = -5/16. 3 is the largest, so 3 is our answer.

Hint: Just calculate each value! Pretty straightforward!

10. Brian the Bear has 4 sisters and 6 brothers. His sister Barbie has S sisters and B brothers. What is the product of S and B?

Solution: Brian is a boy, meaning that there are 4 girls and 7 boys in total. This means that Barbie would have 3 other sisters(she can't count herself as a sister) and 7 brothers. This means that S = 3 and B = 7. 3*7 = 21.

Hint: Think about the total number of boys and girls in the household before thinking about the number of sisters and brothers Barbie has.

11. The 10-letter code MATH IS LUCK represents ten digits 0-9, in order. What 4-digit number is represented by the code word HATS?

Solution: Numbering the code in order, we see that H = 3, A = 1, T=2 and S=5. Therefore, our answer is 3125.

Hint: Write out the code, then write the digits in order on top of the letters, so you can see which letters correspond to which number.

12. Express 1/32 as a decimal.

Solution: An easy way to think about this is knowing that 1/32 = 1/8 * 1/4. We know that 1/8 = 0.125(if you don't know this, try further breaking 1/8 down into 1/4 * 1/2). Just multiply 0.125*0.25 to yield 0.03125.

Hint: Consider breaking up 1/32 into a product of other fractions that you can deal with.

13. What is the remainder when 123456789 is divided by 11?

Solution: Using the divisibility rule for 11, we add every other digit: 1+3+5+7+9 = 25, and subtract the other set of digits: 2+4+6+8 = 20, to yield 25-20 = 5.

Hint: You could just divide it...but try using the divisibility rule for 11. If you don't know that, try to derive it, by finding a pattern in the digits of numbers you KNOW are divisible by 11. This makes life much easier.

14. A rectangle has an area of 25 square centimeters. If the length is 1 meter, what is the width in meters?

Solution: The length is 1 meters. However, 1 meter is also 100 cm. Let the width of the rectangle be w. We know that, by the formula for the area of a rectangle: 100*w = 25 w = 0.25 cm. However, we know need to convert this width back to meters. The answer is then 0.0025 m.

Hint: Why not convert the given length into centimeters, then go from there using the area of a rectangle. Remember, it is hard to convert from square meters to square centimeters.

15. What is 2(1-1/2)+3(1-1/3)+4(1-1/4)+ ...+20(1-1/20)?

Solution: First, we write out the first few terms of the sequence to see what we have: 2(1/2) + 3(2/3) + 4(3/4) + ...20(19/20) 1 + 2 + 3+ .....19. Aha! This is a sum that is easy to calculate: The answer should simply be 19*20/2 = 190.

Hint: Multiply out the first few terms of the sequence...see if you get a sum that is easy to calculate!

16. If $n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot .... \cdot (3) \cdot (2) \cdot (1)$, what is $9!/(5! \cdot 4!)$? For example, 4! = 4*3*2 = 24. This notation is referred to as a factorial, and 4! is pronounced "four factorial."

Solution: Answer should just be (9*8*7*6)/(4*3*2*1) = 126.

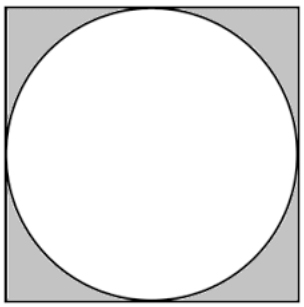
Hint: Don't calculate each factorial. Write each one out and see if anything cancels out to make the division easier.

17. Boris takes a two digit number, subtracts it from 300, and triples the result. What is the largest number he can get?

Solution: A two-digit number subtracted from a 3-digit number(300 in this case) will always be negative, so we want the smallest possible difference. This will happen with the two-digit number 10. The answer is (10-300)*3 = -870.

Hint: Consider the sign of your answer(positive and negative), and find out what two-digit number will be useful.

18. What is the area of the smallest square that will inscribe a circle of radius 3?



Solution: If we draw the picture out, we see that if we draw the diameter of the circle, from end to end, it is just the same as the side length of the square. The diameter is twice the radius, so the diameter is 6, and the side length is therefore 6. The area of this square is 36.

Hint: Draw the diagram out! Figure out some nice lengths to work with.

19. Let f(x) be a function that is linear, and satisfies f(0) = -4 and f(3) = 2. Find f(f(6)).

Solution: We can calculate the slope of the line that passes through these two points. It should be 0-3/(-4-2) = -3/-6 = 1/2. Because we know the y-intercept is f(0) -4, we know the equation of the line. The equation of the line is y = x/2 - 4. f(f(x)) means that we plug in x/2 - 4 into x for the function. This is:

(x/2- 4)/2 - 4 = (x-8)/4 -4 = (x-24)/4. Plugging in 6 for this yields -18/4 = -4.5.

Hint: Try to solve for the equation of the line, since the problem gives you the function as linear.

20. Evaluate the following expression: $\sqrt{20+\sqrt{20+\sqrt{20+...}}}+\sqrt{30+\sqrt{30+\sqrt{30+...}}}+\sqrt{42+\sqrt{42+\sqrt{42+...}}}$

Solution: Let's take this one step at a time, by evaluating the first nested square roots to begin with: Let $x = \sqrt{20+\sqrt{20+\sqrt{20+....}}}$. Squaring this equation, we get: $x^2 = 20+\sqrt{20+\sqrt{20+\sqrt{20+...}}}$ Aha! We now have the following: $x^2 = 20 + x$. This is just a simple quadratic equation, and moving terms and factoring yields: $(x-5)(x+4) = 0$ So we know that x = 5. I will leave the rest of the square root calculations as an exercise for the reader. The answer you should get is: 5 + 6 + 7 = 18.

Hint: Try breaking the problem up by solving one square root at a time. Additionally, let the square root you are solving for equal a dummy variable, then see if you can manipulate the equation somehow to get a nicer equation that you can work with.