

The 1st Annual West Windsor-Plainsboro Mathematics Expo

Saturday, October 26th, 2019

Grade 3 Problem Set

Directions:

Solve the following problems to the best of your ability. If you do not understand a problem or cannot solve it, skip it or ask for a hint. If you cannot solve a problem even after receiving all the hints for that problem, wait until the 30 minute mark and ask a proctor for further help or the solution. Some problems may not have hints.

Calculators are not allowed for these problems. You may, however, discuss with the people around you after 30 minutes have passed. That being said, do not ruin a problem for somebody by giving them a solution before they have a chance to attempt the problem themselves.

For this test, there will be 20 questions, and you will have a time limit of 60 minutes in total, which will be split into 30 minutes of individual work and 30 minutes of collaborative work. This test is very long and you are not expected to be able to do all of the problems. We recommend picking a range of 10-15 problems to work on.

Please note that this is not a competition, and your goal is to enjoy the problems and gain experience.

HAVE FUN!

By the way, if you finish this exceptionally early, you are most likely an exceptional student. Thus, here is a slightly harder problem that you may wish to solve:

CHALLENGE: Find all values of x less than 1000 that satisfy the following modular equations:

$$x \equiv 1 \pmod{2}$$

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 4 \pmod{7}$$

$$x \equiv 5 \pmod{11}$$

For those who do not understand the notation, this simply means that x leaves a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 5, a remainder of 4 when divided by 7, and a remainder of 5 when divided by 11.

1. What is $2 + 2$?

○ ○
○ ○

Solution: Simple addition: 4

Hint: Add.

2. 0 is this problem writer's second favorite number. What is $0 + 0$?

Solution: 0

Hint: Add.

3. What is $2 + 0 + 2 + 2$?

Solution: 6

Hint: Add.

4. Mickey Mouse has 6 marbles. Donald Duck has 8 marbles. Goofy has 53 mangoes. How many marbles do they have all together?



Solution: $6 + 8 = \boxed{14}$

Hint: Use addition. By the way, Halloween is coming up so trick or treat!

5. Thomas the Tank Engine has 3 years until he graduates Train School. If he has to be 18 to graduate, how old is he?

Solution: Since he has to be 18 to graduate, we know that he is 3 less than 18 years old. Thus, he is $18 - 3 = \boxed{15}$ years old.

Hint: Use subtraction.

6. Andrew has 50 stars. Hurricane Jyotika destroyed 32 of them. How many stars does Andrew have left?

Solution: $50 - 32 = 18$.

Hint: Use subtraction.

7. What is $1 + 1 + 1 + 1 + 1 - 1 - 1 - 1 - 1 - 1$?

Solution: Either add consecutively or notice that there are 5 added ones and 5 subtracted ones which cancel out to 0

Hint: Add and subtract.

8. What is $1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4)$?

Solution: $1 + 3 + 6 + 10 = 20$

Hint: Add. If they still can't get it, tell them that the first digit of the answer is the author's first favorite number and the second digit of the answer is the author's second favorite number.

9. Continue the pattern: 1, 4, 7, 10, ---.

Solution: Notice that this is a sequence where terms differ by 3, so the next number is 13

Hint: Subtract each number from the number after it.

10. What is the next number in this sequence of numbers, 1, 3, 6, 10, 15, 21, -- ?

Solution: Note that these are triangular numbers and pray that the sequence is of triangular numbers. The next triangular number is 28. (Helper will need to explain the definition of a triangular number)

Hint: What is 1? What is $1 + 2$? What about $1 + 2 + 3$? Hmm...

11. 2 is this problem writer's favorite number. What is 2×2 ?

Solution: $2 \times 2 = 4$

Hint: Explain what multiplication is.

12. There are about 2000 kids in high school. If half of them wear blue shirts and 1024 of them have blue hats, what is the least number of people that both wear blue shirts and have blue hats?



Solution: We know that $\frac{1}{2}$ or 1000 of them wear blue shirts, so the least number of people that wear both is simply $1024 - 1000$ or $\boxed{24}$.

Hint: Draw a Venn Diagram.

13. Ayush has ten stickers. He dropped his orange juice over half of them and now those stickers cannot be used. How many stickers can be used?

Solution: $10 - 5 = \boxed{5}$.

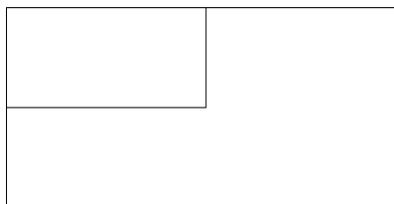
Hint: Use subtraction.

14. 4 integers from a set of 5 are 5, 7, 2, 3. What is the maximum possible median of this set?

Solution: When listed in ascending order, this is 2, 3, 5, 7. If the other number is less than or equal to 3, the median will be 3. If it is four, the median will be 4. If it is greater than or equal to 5, the median will be 5. Thus, the largest median is $\boxed{5}$.

Hint: What are the possible medians?

15. If the lengths of a rectangle are doubled, then how many times greater is the area of the new rectangle than the area of the original rectangle?



Solution: Since the area is the length times the width, if they are both doubled, the area is $2 \times 2 = \boxed{4}$ times greater.

Hint: Use the diagram.

16. Bob goes to Costco and buys five 72-pound wheels of cheese (which Costco actually sells). How many pounds total?

Solution: $72 + 72 + 72 + 72 + 72 = 5 \times 72 = \boxed{360}$.

Hint: Draw a diagram.

17. $a * b = 2a + 2b$. What is $5 * 3$?

Solution: $2 \times 5 + 2 \times 3 = 10 + 6 = \boxed{16}$.

Hint: Substitute the numbers into the expression.

- 18.

$$\triangle + \square = 4$$

$$\circ + \square = 6$$

$$\triangle + \circ = 8$$

Find $\triangle + \square + \circ$.

Solution: By adding the three equations, one finds that:

$$2\triangle + 2\square + 2\circ = 18$$

This means that

$$\triangle + \square + \circ = \boxed{9}$$

Hint: Add the equations together.

19. A right triangle has two legs of lengths 3 and 4. What is its area?

Solution: Since it is a right triangle, we can consider 3 to be the base, and the height must be 4. Thus, the area is $\frac{1}{2} \cdot 3 \cdot 4 = \boxed{6}$.

Hint: Draw a picture of a right triangle.

20. Solve for x using Algebra!

$$x \cdot x = x + x$$

Please note that \cdot is the same thing as \times .

Solution: First, note that $x = 0$ is a solution. Now, we can divide by x to find the solutions that are not zero:

$$x = 1 + 1$$

Thus, the other solution is:

$$x = 2$$

The two solutions are $\boxed{0 \text{ and } 2}$.

Hint: Divide by x , but don't forget something.