

# The 1st Annual West Windsor-Plainsboro Mathematics Expo

Saturday, October 26<sup>th</sup>, 2019

## Grade 8 Problem Set

### Directions:

Solve the following problems to the best of your ability. If you do not understand a problem or cannot solve it, skip it or ask for a hint. If you cannot solve a problem even after receiving all the hints for that problem, wait until the 30 minute mark and ask a proctor for further help or the solution. Some problems may not have hints.

Calculators are not allowed for these problems. You may, however, discuss with the people around you after 30 minutes have passed. That being said, do not ruin a problem for somebody by giving them a solution before they have a chance to attempt the problem themselves.

For this test, there will be 20 questions, and you will have a time limit of 60 minutes in total, which will be split into 30 minutes of individual work and 30 minutes of collaborative work. This test is very long and you are not expected to be able to do all of the problems. We recommend picking a range of 10-15 problems to work on.

Please note that this is not a competition, and your goal is to enjoy the problems and gain experience.

### ***HAVE FUN!***

By the way, if you finish this exceptionally early, you are most likely an exceptional student. Thus, here is a slightly harder problem that you may wish to solve:

### **CHALLENGE:**

$$\left( \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} \right)^{-1}$$

1. Evaluate:  $2 + 2 \times 2$

2. Define  $A(a, b)$

$$A(a, b) = \begin{cases} b+1 & \text{if } a = 0 \\ A(a-1, 1) & \text{if } a > 0 \text{ and } b = 0 \\ A(a-1, A(a, b-1)) & \text{if } a > 0 \text{ and } b > 0 \end{cases}$$

a) Find  $A(0, 2)$

b) Find  $A(1, 2)$

c) Find  $A(2, 2)$

d) Find  $A(3, 2)$

3. Suraj has a lot of debt. He has worked out a payment plan where he pays \$5 every day in the morning, but at the end of each day if he still has debt, \$2 of interest will be added. If he currently has \$25 of debt and the day has just started (has not paid morning payment), how many days will it take to get rid of all the debt?



4. David the bear loves to break down wire fences that are used to keep bears out. He has two wires of the same length. He creates an equilateral triangle from the first one, and he creates a square from the second one. Find the ratio of the area of these 2 shapes. (triangle to square)



5. Five cows are in a line. Farmer John wants to paint each of them 1 of 3 colors, but if two cows next to each other are both the same color he gets confused. Find the number of ways he can paint his cows without getting confused.

6. a) Let  $m$  and  $n$  be positive integers such that  $3m + 2n = 2019$ . How many values of  $m$  and  $n$  are there?

b) Let  $m$  and  $n$  be positive integers such that  $3m + 2n \leq 2019$ . How many values of  $m$  and  $n$  are there?

7. Kagami takes 1 hour to walk from her house to her school. She takes the bus on the way home which only takes 20 minutes. Given that her average speed is 9 km/hr, how far away is the school in km?



8. Adi sucks at handling money. He currently has \$20. Someone went up to him and said, "I will increase your money by 10%, but then I would take 10% away from you". Adi agreed, what is the difference between amount of money he had before and the amount of money he has now?

9. Inazuma and Ikazuchi both went to an ice-cream stop on a certain day between 2:00 pm and 3:00 pm, staying for a total of 10 minutes each. What is the probability that they were able to meet up?

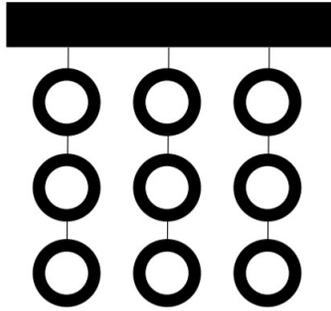


10. Circle  $O_0$  and circle  $O_1$  intersect at  $A$  and  $B$ . Given that the radius of circle  $O_0$  is 24 and the radius of circle  $O_1$  is  $24\sqrt{3}$ , and  $\overline{AB}$  is  $24\sqrt{3}$ , find the area of the intersection of the circles.
11. Let  $N = 2019^{10}$ . How many factors of  $N^2$  less than  $N$  are there that do not divide  $N$ .
12. Violet is randomly typing keys on her typewriter. There are 45 keys, 25 of which are letters and the rest are punctuation. She randomly types out 7 characters. What is the probability that she just typed 2 different 3-lettered "words" (only letters) with a punctuation in between. For example, "AAA.BBB" is one such possibility.
13. Pratyoy Biswas is forced to watch Death Note! Given that each day there is a 50% chance he is too busy, 25% chance he watches an episode, and a 25% chance he watches two episodes (if only one episode remains he only watches one episode). If Death Note is a 5 episode show, what is the expected number of days to watch the entire thing?



14. Let  $a, b, c$  be the roots of the polynomial  $x^3 - 5x^2 + 9x - 12$ . Find  $a^2 + b^2 + c^2$ .

15. Verniy is shooting targets. She has to shoot 3 strings of 3 targets. For each string, the targets are arranged vertically such that hitting the top target will make the whole column fall and hitting the middle would make the middle and bottom targets fall. How many ways can she shoot at the targets such that no target remains after she is done? Verniy is amazing at shooting and does not miss. The order in which Verniy takes the shots matters.



16. How many non-empty subsets  $S$  of the set  $\{1, 2, 3, \dots, 15\}$  have the following two properties?
- (1) No two consecutive integers belong to  $S$ .
  - (2) If  $S$  contains  $k$  elements, then  $S$  contains no number less than  $k$ .
17. a) An ant is currently at  $(0, 0)$  and is trying to get to its base at  $(5, 5)$ . There is currently a wall that is the line  $y = x$ . If it can not cross this wall, but can be on it, how many ways can it get to  $(5, 5)$  given that it can only move one unit right or one unit up at a time.
- b) How many ways are there to write out 5 pairs of parentheses such that it is a valid combination of parentheses? Ex.  $((((()())))$  or  $((()()()))$  not  $((()())()$  or  $))((()())$
18. There is a naval base that is 12 docks side by side. An aircraft carrier would require 3 docks, a cruiser would require 2 and a small submarine would require 1. Given an infinite amount of all types of ships, and all ships of the same class are indistinguishable, how many ways are there to fill all the docks of the naval base?
19. How many decreasing sequences  $a_1, a_2, \dots, a_{2019}$  of positive integers are there such that  $a_1 \leq 2019^2$  and  $a_n + n$  is even for each  $1 \leq n \leq 2019$ ?
20. Let  $a, b$  be positive real numbers with  $a > b$ . Compute the minimum possible value of the expression  $\frac{a^2b - ab^2 + 8}{ab - b^2}$